Solutions to Introductory Tutorials Thermodynamics Project 2019

Exercise 1

First of all we need to realize that the thermal conductivity of the bath has to be infinite in order for the transfer of heat to be ideal. We disregard whether or not this is realistic.
Furthermore, we work at a constant volume ($\Delta V = 0$) so that the work is $W = P\Delta V = 0$, and $\Delta U = Q$.

a) $\Delta U = c_V \rho V \Delta T = 4.2 \cdot 1.0 \cdot 10^6 \cdot 100 \cdot 10^{-6} \cdot (323 - 300) = 9.7$ kJ.

b) For the final temperature of the bath, we neglect the volume of the tube.
$\Delta T_{bath} \approx \frac{\Delta U}{c_V \rho V_{bath}} = \frac{c_V \rho V_{tube} \Delta T_{tube}}{c_V \rho V_{bath}} = \frac{V_{tube}}{V_{bad}} \Delta T_{tube} = \frac{100 \cdot 10^{-3}}{100} \cdot (323 - 300) = 0.023$ K.
So the final temperature of the bath is approximately $T_{bath}^{final} \approx 300.023$ K.

c) $V_{bath} = 10000$ l: $T_{bath}^{final} = 300.00023$ K.
$V_{bath} = 100$ ml: $T_{bath}^{final} = 323$ K.
The approximations in a) and b) are of course completely inaccurate for the bath of 100 mL. For that case, we use a more accurate method.
$\left(T_{bath} - T^{final}\right) = \frac{\Delta U}{c_V \rho V_{bath}}$ and $\left(T_{tube} - T^{final}\right) = \frac{-\Delta U}{c_V \rho V_{tube}}$, so
$\left(T_{bath} - T^{final}\right) V_{bath} = \left(T_{tube} - T^{final}\right) V_{tube}$, or $T^{final} = \frac{T_{bath} V_{bath} + T_{tube} V_{tube}}{V_{bath} + V_{tube}}$.
Which gives for $V_{bath} = 100$ ml: $T^{final} = 311.5$ K.

d) The 100 L bath seems reasonable; the 10000 is even better.
The choice depends on the desired accuracy of your experiment.

Exercise 2

a) No work is done on the water. The mechanical work is zero; the vial is completely filled and enclosed and therefore a change in volume cannot occur upon heating. Only electrical work is done on the resistor.

b) The current I, the duration of the measurement Δt (the time for a running current).

c) The electrical power dissipated in the resistor is $P = I^2 R$.
The corresponding electrical energy (work) is $W = P\Delta t$.
Since the calorimeter is well isolated, $Q = 0$ applies so $\Delta U = W$.
The change in temperature is thus $\Delta T = \frac{W}{c_V \rho V} = \frac{I^2 R \Delta t}{c_V \rho V}$, or $I^2 \Delta t = \frac{\Delta T c_V \rho V}{R}$.
For the resolution to be ± 0.1 °C, we need to make sure that $I^2 \Delta t = \frac{0.1 \cdot 4.2 \cdot 1.0 \cdot 10^6 \cdot 250 \cdot 10^{-6}}{10}$ with I in Ampere and Δt in seconds. Or $I^2 \Delta t = 10.5\ \mathrm{A^2 s}$.

d) For a measurement of 60 seconds we need a current of approximately 400 mA to reach the resolution of the thermometer. Those values are quite realistic.
A true test for the first law of thermodynamics would include multiple measurements with different durations and/or currents.
However, the temperature range is limited and the amount of testing depends on the accuracy that is needed.
Furthermore, only electrical work is considered in this exercise.