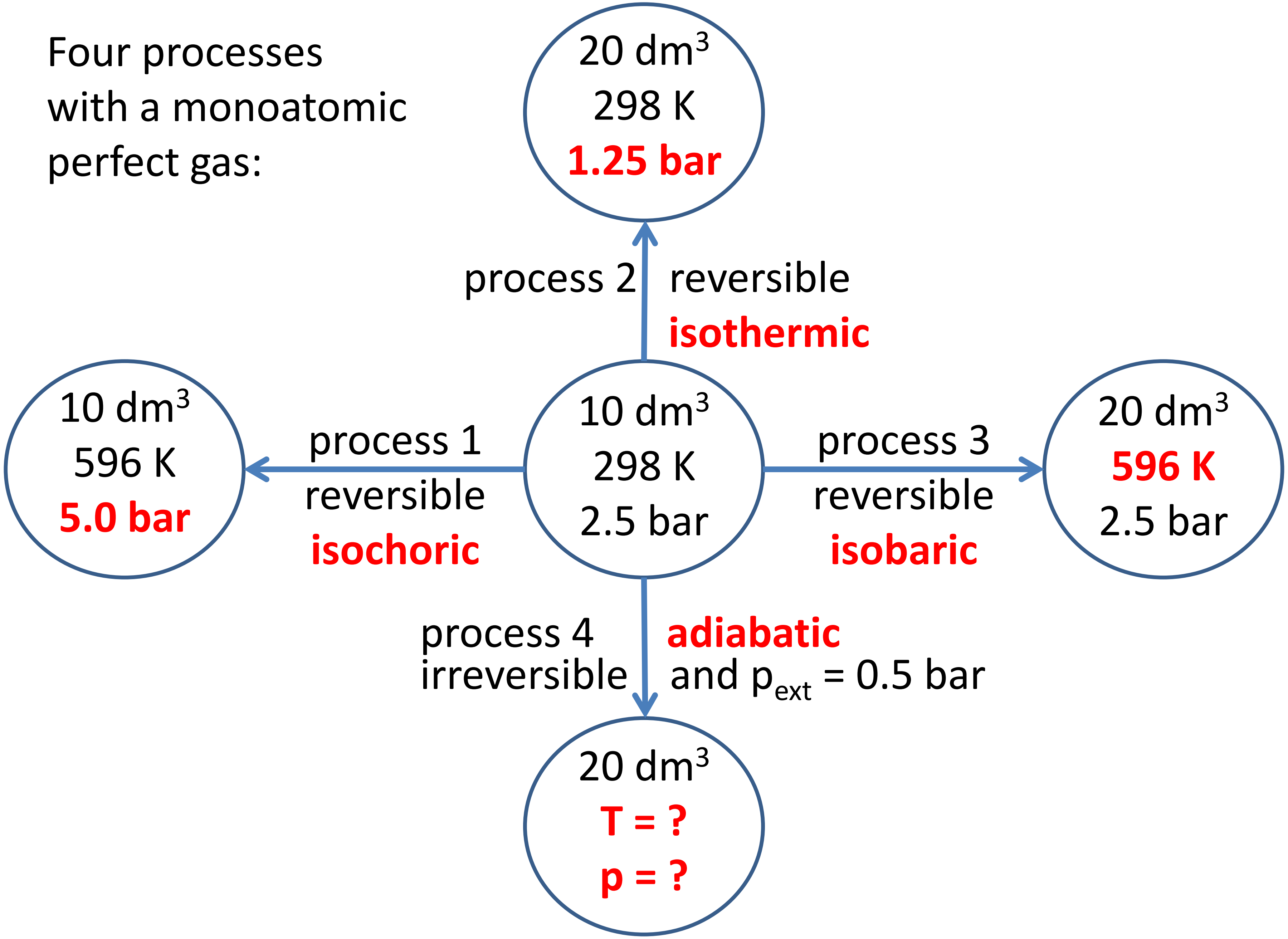
Thermodynamics tutorhour 2

Four processes with a monoatomic perfect gas:



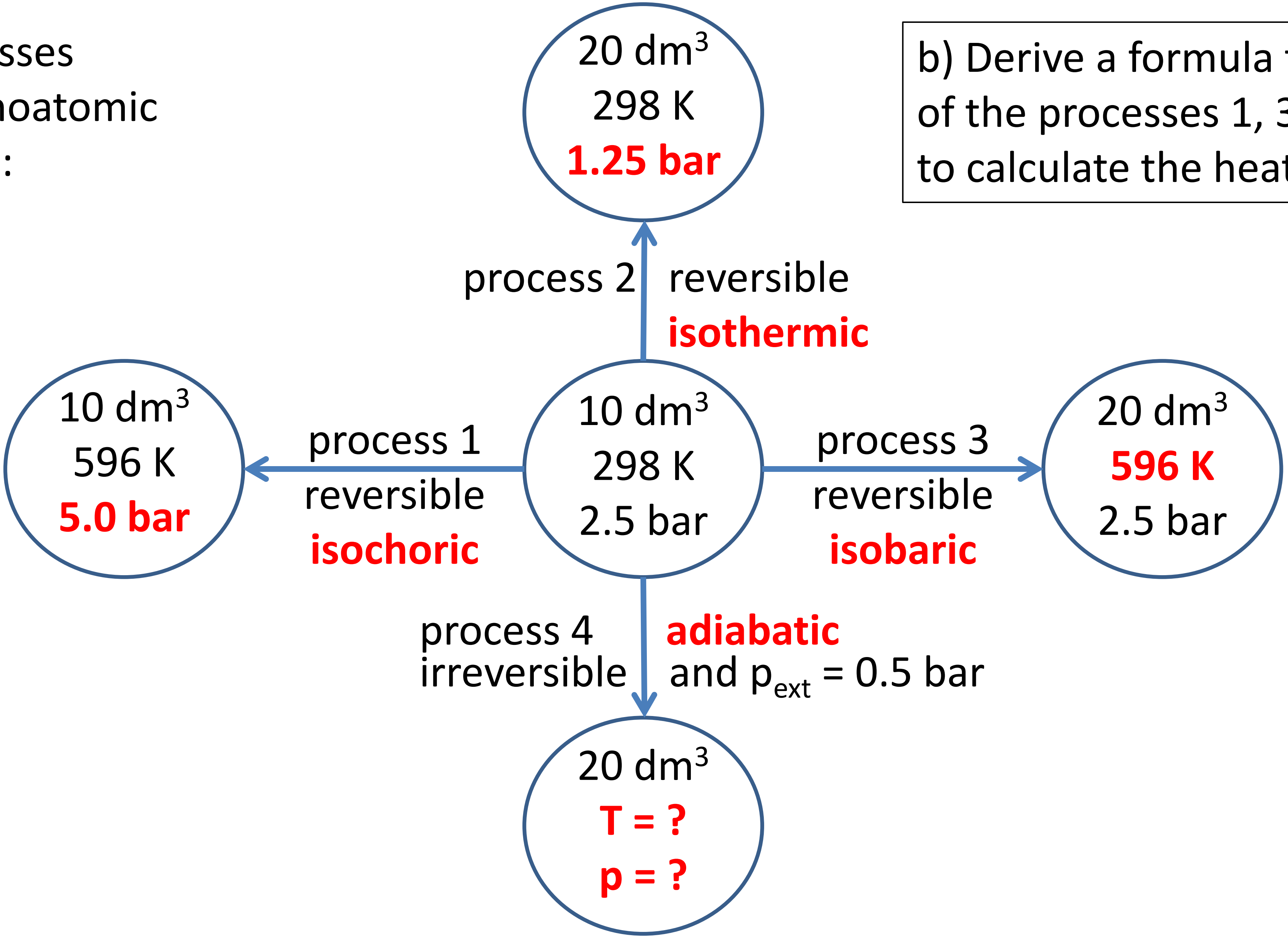
a) Derive a formula to calculate the exerted work W for each of the four processes, using the formula:

$$W = -\int p\mathrm{d}V \qquad \text{Perfect gas law: } pV = n\mathrm{R}T; \quad p = \frac{n\mathrm{R}T}{V}$$

No	kind of process	*W*	*Q*	*ΔU*
1	isochoric	**0**		
2	isothermic	$-n\mathrm{R}T \cdot \ln(V_f/V_i)$		
3	isobaric	$-p_{ext}\Delta V$		
4	adiabatic	$-p'_{ext}\Delta V$		

Four processes with a monoatomic perfect gas:

b) Derive a formula for each of the processes 1, 3 and 4 to calculate the heat Q.



b) Heat Q: / c) ΔU

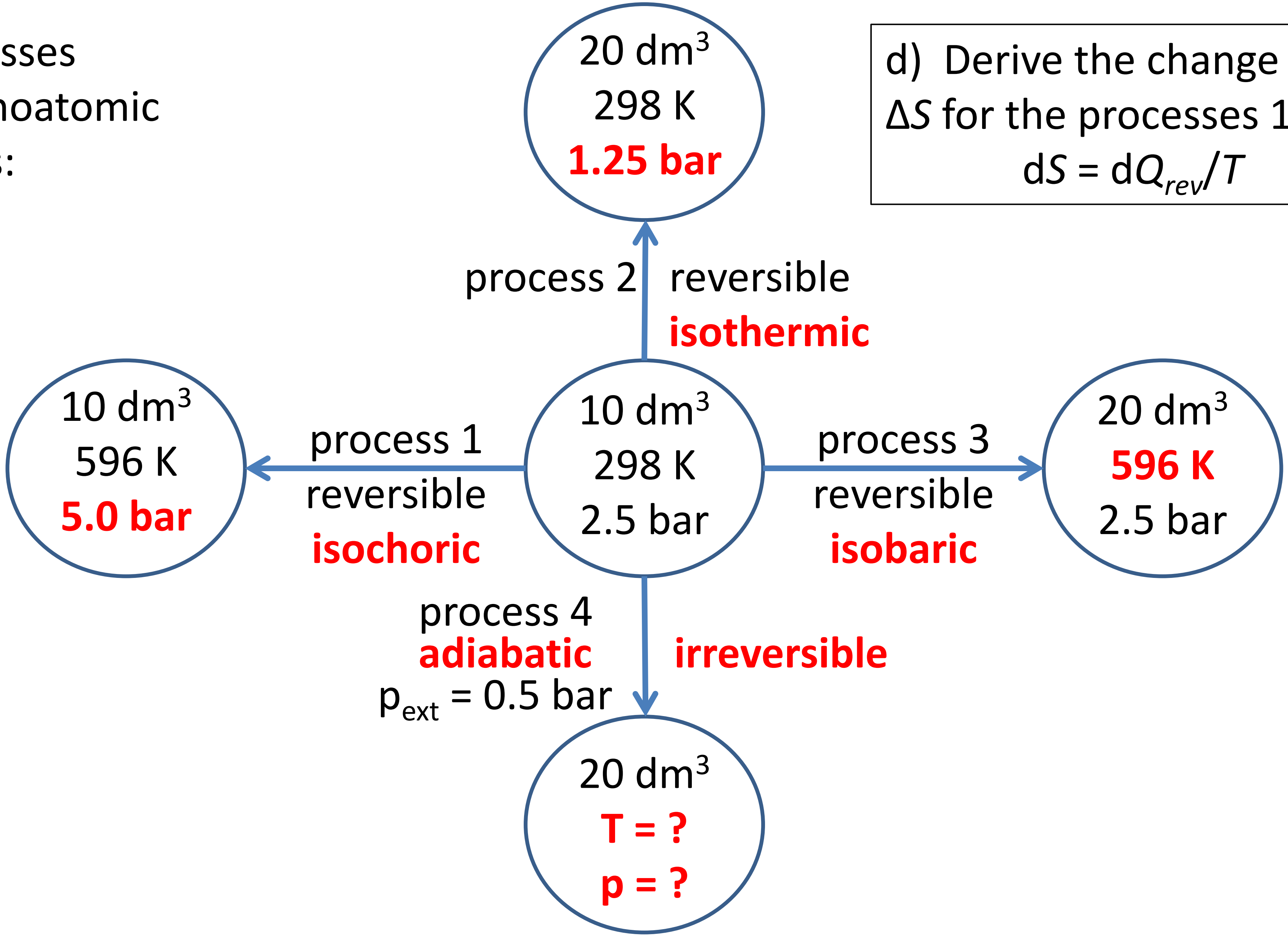
For a **monoatomic perfect gas** is defined: $U = 3/2 \cdot nRT$ so $\Delta U = 3/2 \cdot nR\Delta T$

and: $C_V = 3/2 \cdot nR$

and: $C_p = 5/2 \cdot nR$

Nr	kind of process	W +	Q =	ΔU
1	isochoric	0	$3/2 \cdot nR\Delta T$ / $C_V dT$ / ?	$3/2 \cdot nR\Delta T$ / $C_V dT$
2	isothermic	$-nRT \cdot \ln(V_f/V_i)$	$+nRT \cdot \ln(V_f/V_i)$ / ?	0
3	isobaric	$-p_{ext}\Delta V$ / $-nR\Delta T$	$5/2 \cdot nR\Delta T$ / $C_p dT$ / ?	$5/2 \cdot nR\Delta T - p\Delta V$ / $3/2 \cdot nR\Delta T$ / $C_p dT$
4	adiabatic	$-p'_{ext}\Delta V$	0	$-p'_{ext}\Delta V$

Four processes with a monoatomic perfect gas:



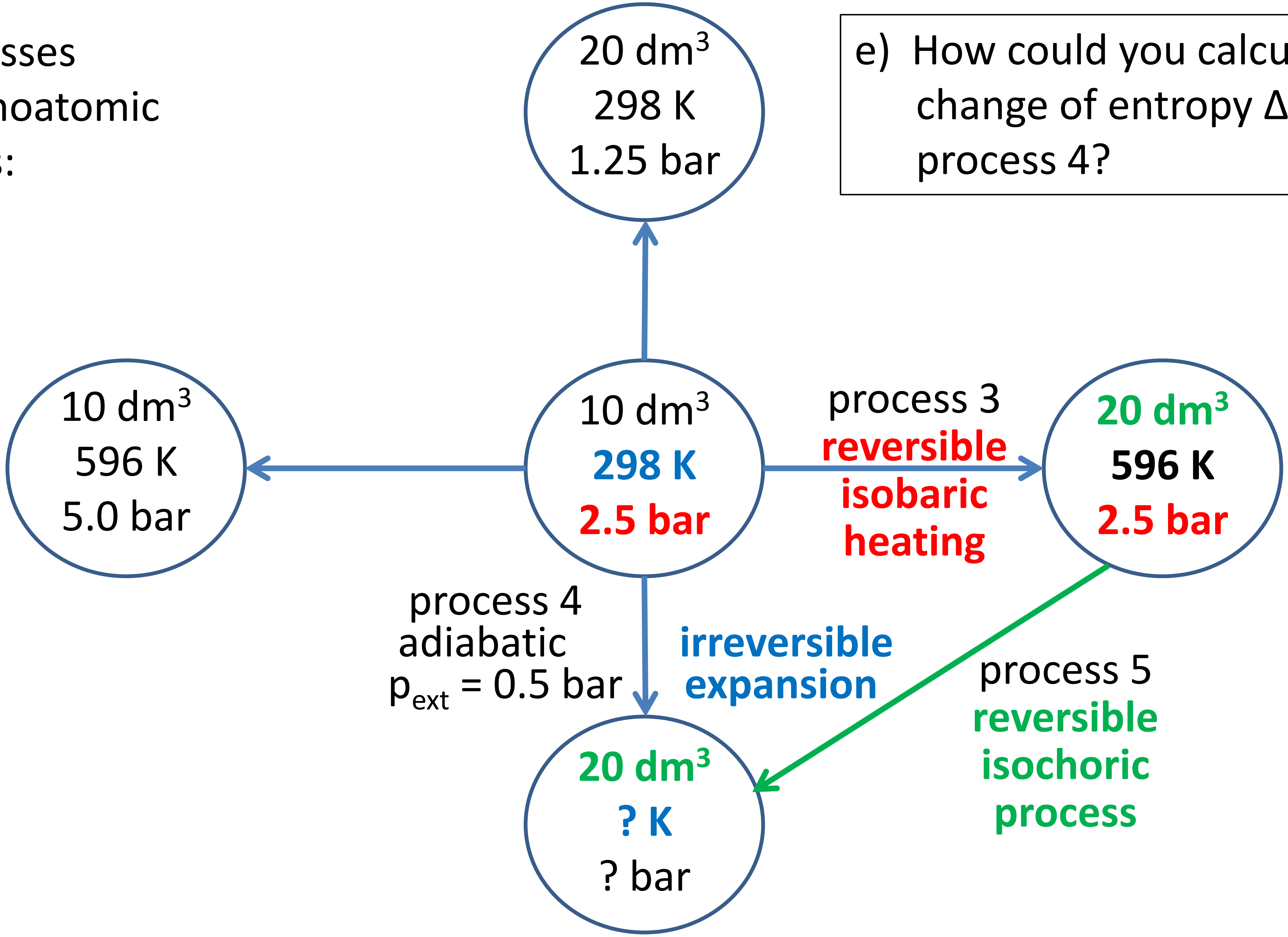
d) Derive the change of entropy ΔS for the processes 1, 2, 3 using:

$$dS = dQ_{rev}/T$$

Nr	kind of process	Q	$\int dQ_{rev}/T$	$= \quad \Delta S$
1	isochoric	$\int C_v dT$	$\int (C_v/T) dT$	$C_v \cdot \ln(T_f/T_i)$
2	isothermic	$nRT \cdot \ln(V_f/V_i)$	⟶	$nR \cdot \ln(V_f/V_i)$
3	isobaric	$\int C_p dT$	$\int (C_p/T) dT$	$C_p \cdot \ln(T_f/T_i)$
4	adiabatic	0	e) irreversible process: you *have to* construct an alternative reversible path	

Four processes with a monoatomic perfect gas:

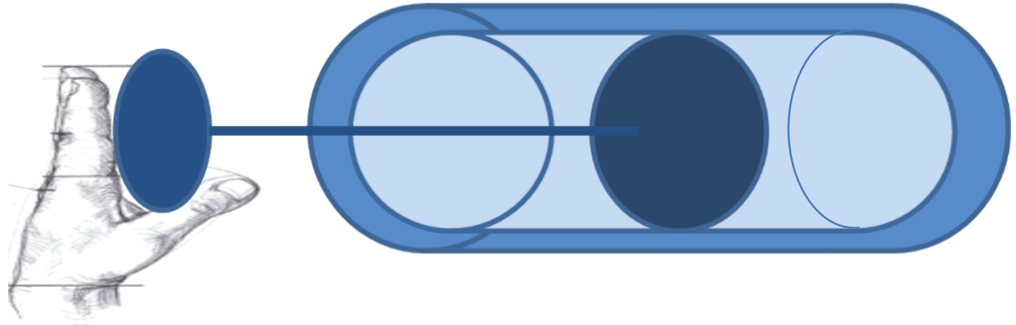
e) How could you calculate the change of entropy ΔS for process 4?



Question 2

We could perform process 4 in the following manner: We take an isolated cylinder with a frictionless piston. On the right hand of the piston there is a monoatomic perfect gas, p = 2.5 bar, and on the left of the piston there is a p_{ext} = 0.5 bar. The initial temperature of the gas is 298 K. Both compartments measure 10 dm^3.

After withdrawing the hand, the piston moves to its maximum volume: 20 dm^3.



a) Calculate the number of moles of the gas present in the cylinder.
b) Calculate the temperature and the pressure when the maximum volume is reached. Use $\Delta U = -p_{ext}\Delta V = 3/2 nR\Delta T$.
c) Calculate the change of entropy, ΔS, of the system.

Question 2

a) $pV = n\mathrm{R}T$

$n = pV/\mathrm{R}T = (2.5 \cdot 10^5 \cdot 10 \cdot 10^{-3})/(8.3145 \cdot 298) = 1.0$ mol

b) $\Delta U = -p_{ext}\, \Delta V = -0.5 \cdot 10^5 \cdot 10 \cdot 10^{-3} = -\ 500$ J

$\Delta U = (3/2)n\mathrm{R}\Delta T = -\ 500$ J; $\quad \Delta T = (-\ 500 \cdot 2)/(3 \cdot 1.0 \cdot 8.3145) = -\ 40$ K

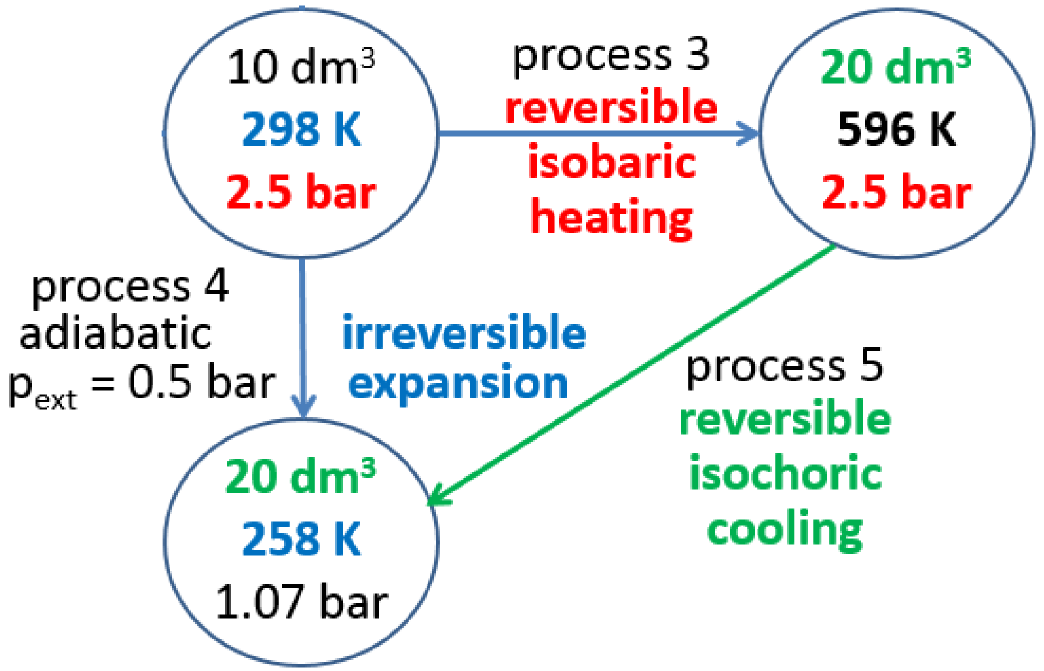
$T_{\text{final}} = 298 - 40 = 258$ K

$p_{\text{system}} = n\mathrm{R}T/V = 1.0 \cdot 8.3145 \cdot 258/0.020 = 1.07 \cdot 10^5$ Pa $= 1.07$ bar

c) We have to construct an alternative <u>reversible</u> path!

Question 2

c) The alternative reversible path for process 4 is successively process 3 and process 5:



$$\Delta S_{process\ 3} = \int dQ_{rev}/T = C_p \cdot \ln(T_f/T_i) = C_p \cdot \ln(596/298) = C_p \cdot \ln(2)$$

$$\Delta S_{process\ 5} = \int dQ_{rev}/T = C_V \cdot \ln(T_f/T_i) = C_V \cdot \ln(258/596) = C_V \cdot \ln(0.4329)$$

$$\Delta S_{process\ 4} = \Delta S_{process\ 3} + \Delta S_{process\ 5}$$

$$= C_p \cdot \ln(2) + C_V \cdot \ln(0.4329) =$$

$$= 5/2 \cdot nR \cdot \ln(2) + 3/2 \cdot nR \cdot \ln(0.4329)$$

$$= 14.41 - 10.44 = 3.97\ J\ K^{-1}$$