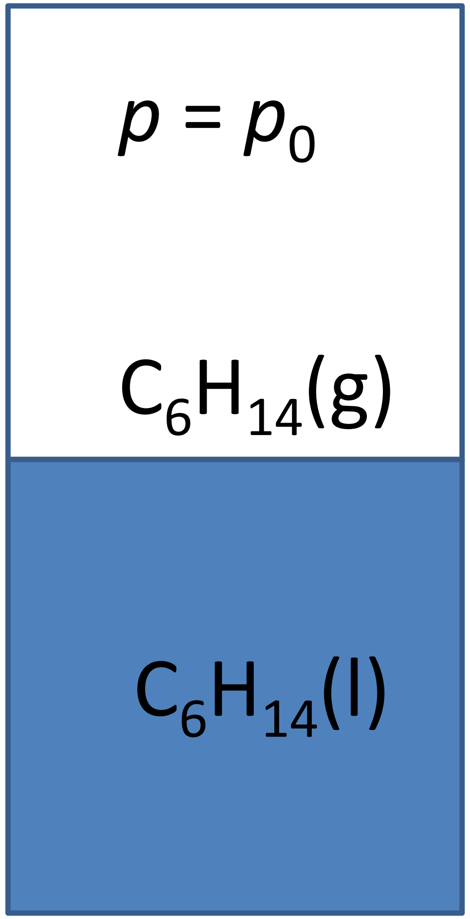
Thermodynamics tutorhour 5

Colligative properties

Elevation of boiling point



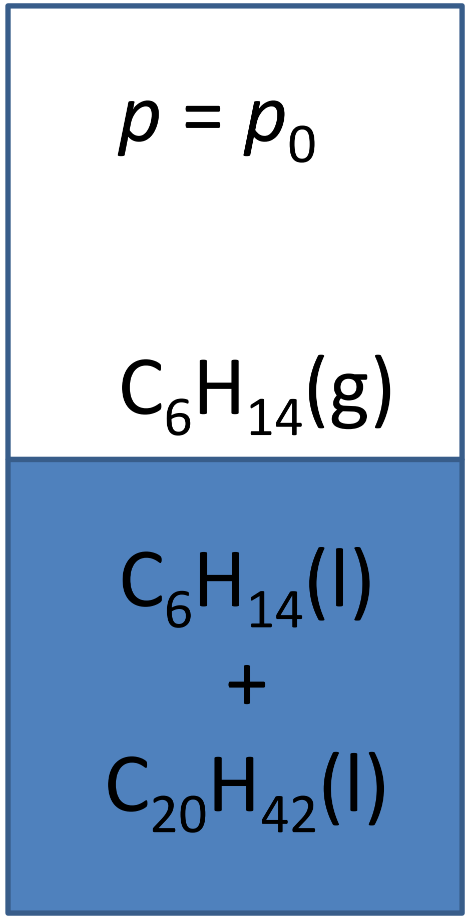
At equilibrium:

$\mu_{gas} = \mu_{liq}$

At $T = T^*_{boil}$ (69^{o}C):

$\mu^*_{gas} = \mu^*_{liq}$

Elevation of boiling point



At equilibrium:

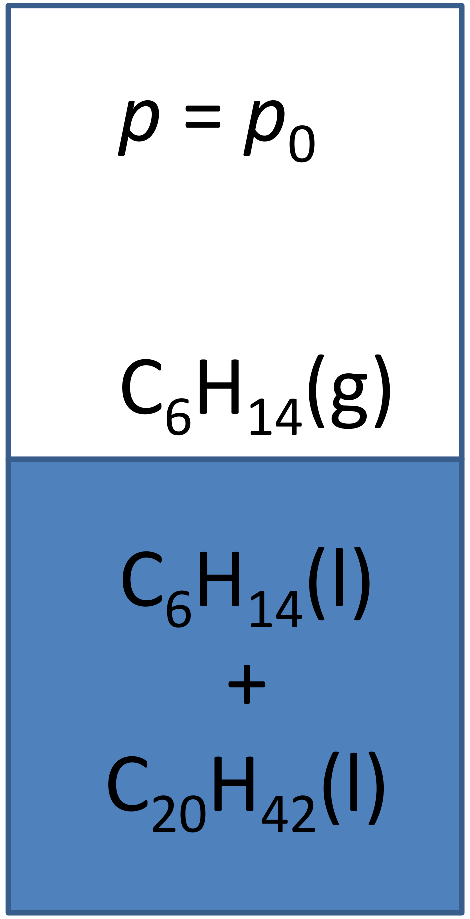
$\mu_{gas} = \mu_{liq}$

$\mu^*_{gas} = \mu^*_{liq}$ + correction for dissolving

$$\Delta T = \left(\frac{RT^{*2}_{boil}}{\Delta_{vap} H}\right) x_B$$

$p = p_0$, ideal dilute solution (low concentration)

Elevation of boiling point



$$\Delta T = \left(\frac{RT_{boil}^{*2}}{\Delta_{vap}H}\right)x_B$$

ΔT : elevation of boiling point

T_{boil}^{*} : boiling point of pure **hexane**

$\Delta_{vap}H$: enthalpy of vaporization of **hexane**

x_B : mole fraction of **eicosane**

$$x_B = \frac{\#\,\text{mol eicosane}}{\#\,\text{mol hexane} + \#\,\text{mol eicosane}}$$

***$p = p_0$* , ideal dilute solution**

The depression of freezing point

$$\Delta T = \left(\frac{RT_{fus}^{*2}}{\Delta_{fus}H}\right)x_B$$

ΔT : depression of freezing point

T_{fus}^{*} : freezing point of the pure **solvent**

$\Delta_{fus}H$: enthalpy of fusion of the **solvent**

x_B : mole fraction of the dissolved **solute**

***p* = *p*$_0$, ideal dilute solution**

Boiling point **elevation** and freezing point **depression**

$$\Delta T = (\frac{RT_{boil}^{*2}}{\Delta_{vap} H}) x_B \qquad \Delta T = (\frac{RT_{fus}^{*2}}{\Delta_{fus} H}) x_B$$

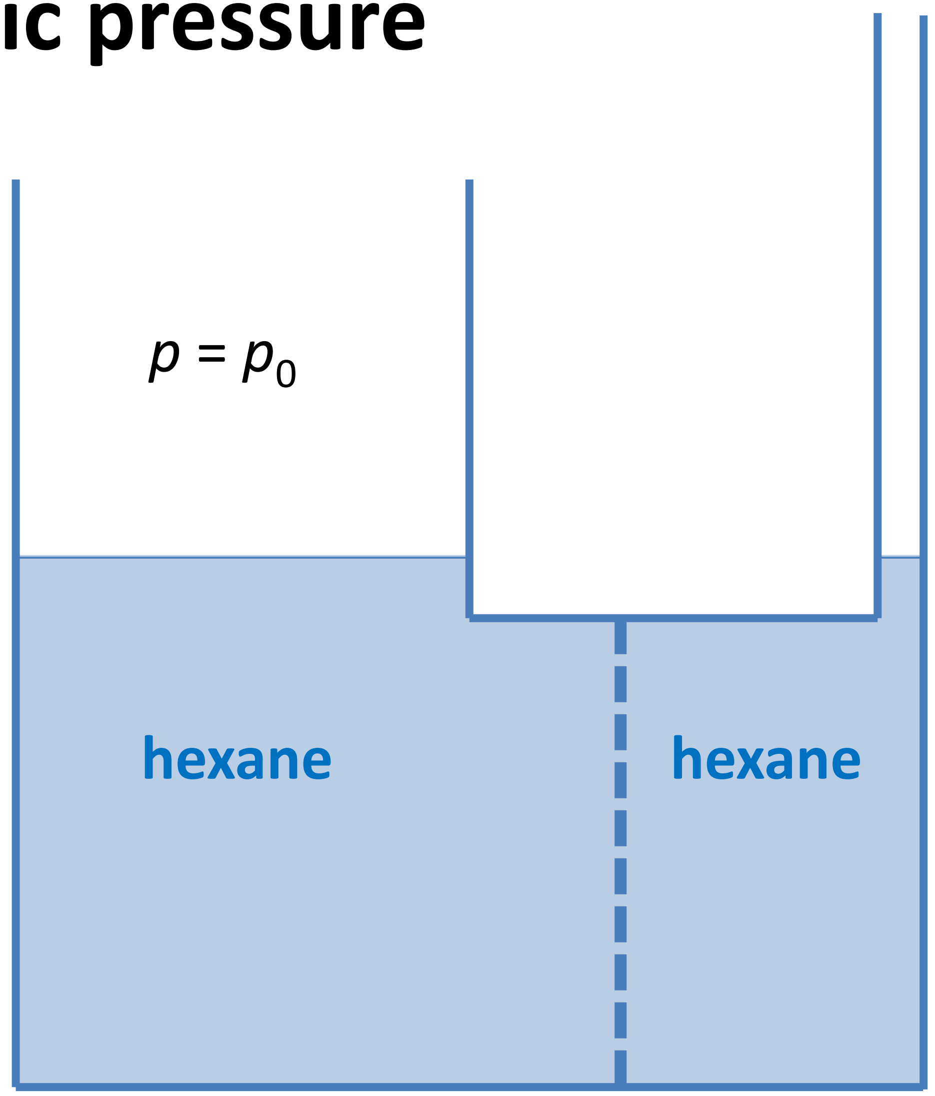
$$\Delta T = \left(\frac{RT^{*2}}{\Delta_{\mathrm{trs}} H} \right) x_B$$

See also the checklist of key equations

trs = transition

$p = p_0$, ideal dilute solution

Osmotic pressure

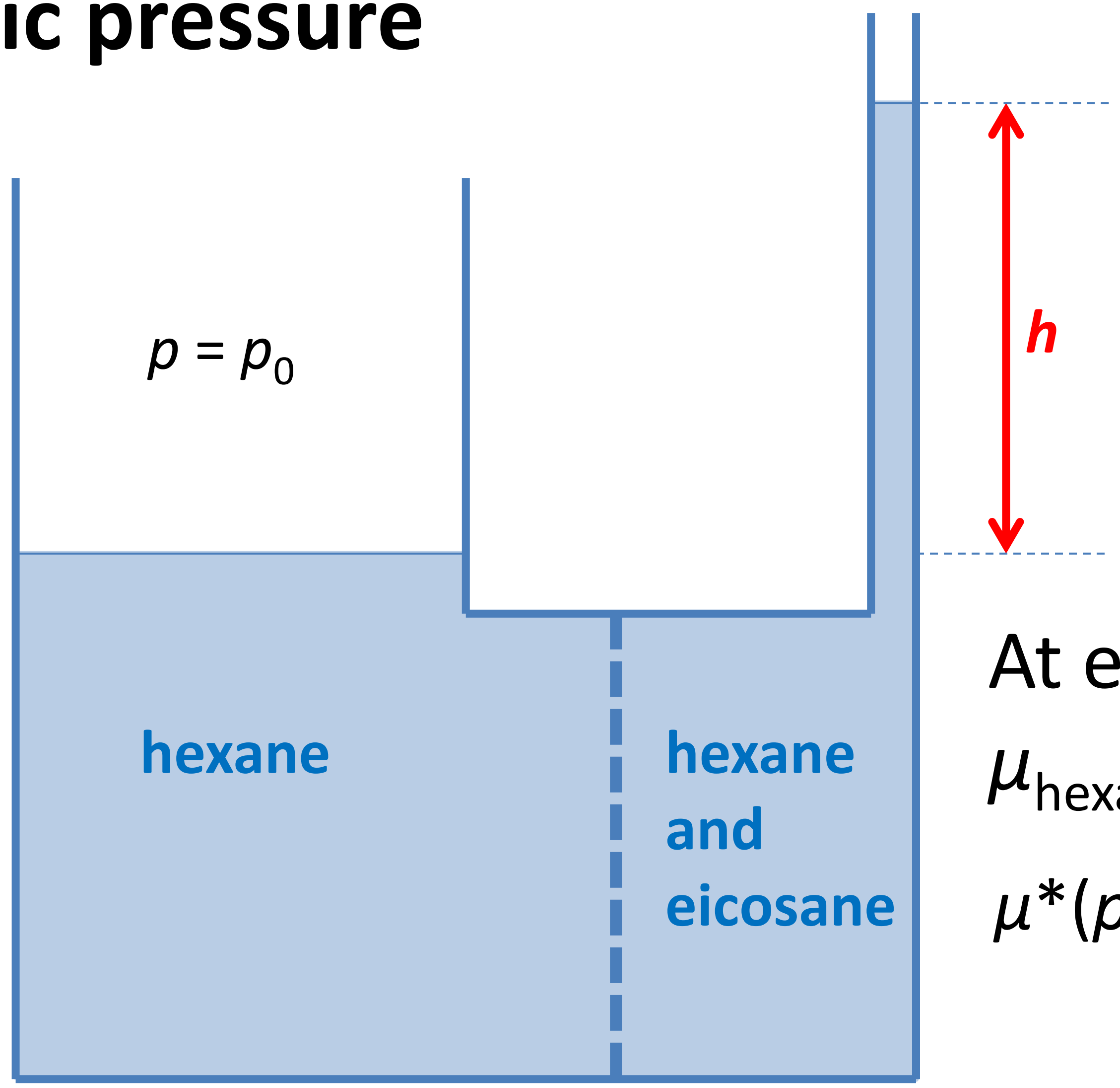


At equilibrium:

$$\mu_{\text{hexane,left}} = \mu_{\text{hexane,right}}$$

$$\mu^*_{\text{hexane}} = \mu^*_{\text{hexane}}$$

Osmotic pressure



additional pressure:

$$p = p_0 + \Pi$$

At equilibrium:

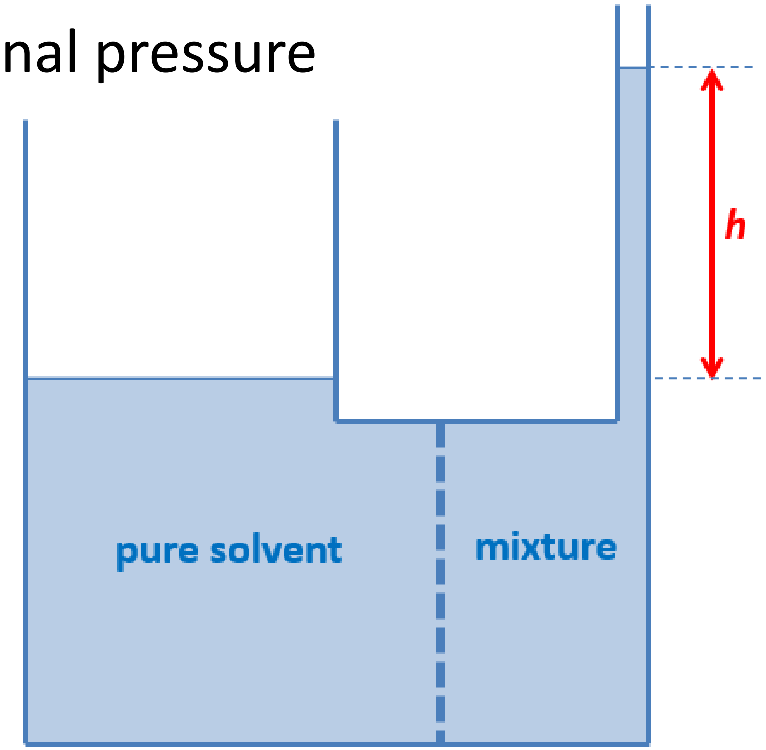
$$\mu_{\text{hexane,left}} = \mu_{\text{hexane,right}}$$

$$\mu^*(p_0)_{\text{hexane}} = \mu(p_0+\Pi)_{\text{mixture}}$$

$$\mu^*(p_0)_{\text{hexane}} = \mu^*(p_0)_{\text{hexane}} + \text{correction for dissolving} + \text{additional pressure}$$

Osmotic pressure

$\mu^*(p_0)_{\text{hexane}} = \mu^*(p_0)_{\text{hexane}}$ + correction for dissolving + additional pressure



***p* = *p*$_0$, ideal dilute solution**

$$\Pi = [B]RT = \frac{n_B}{V}RT$$

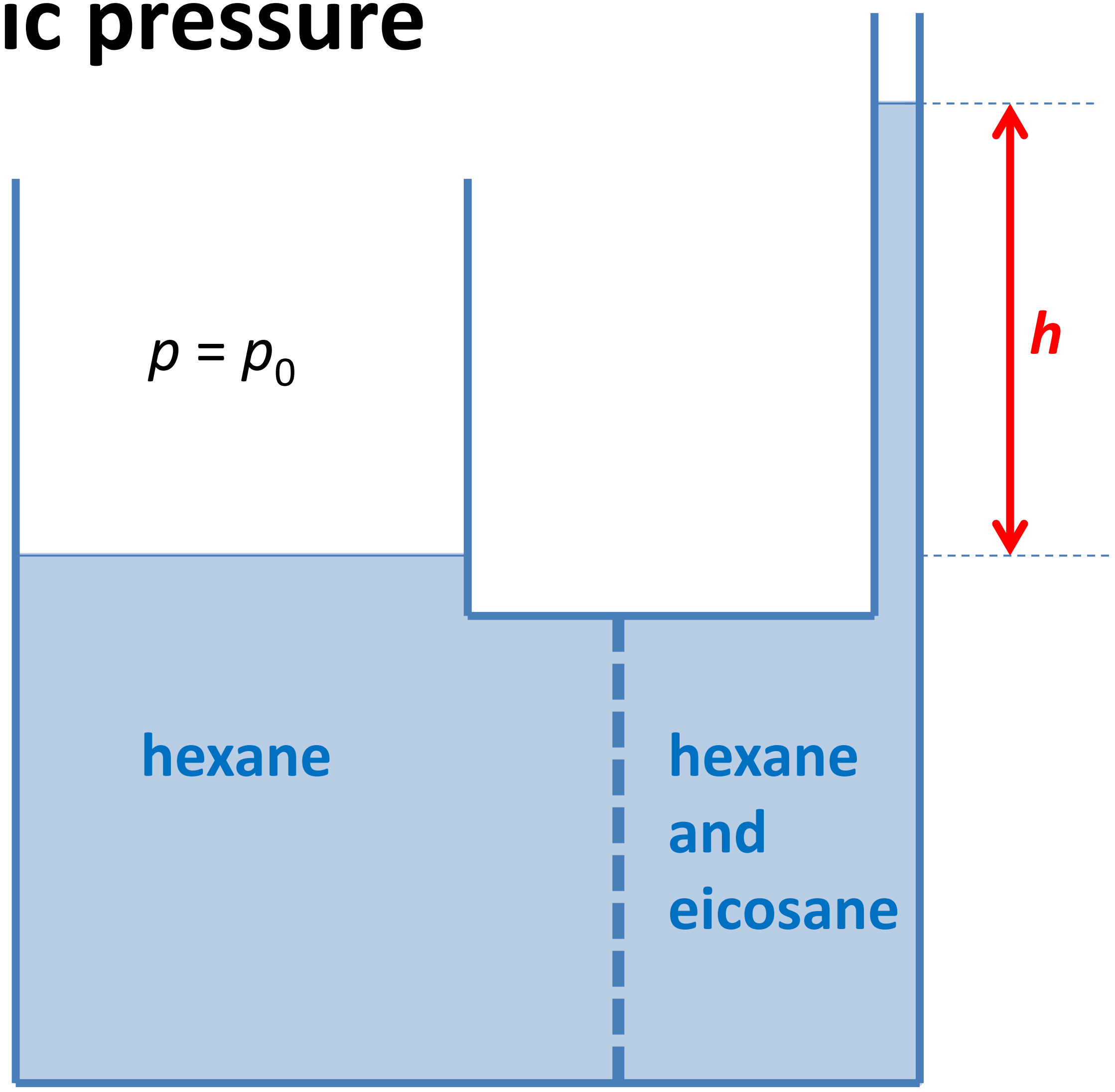
Note: [B] in **mol/m³**

Osmotic pressure can also be expressed as a function of height ***h***:

pressure: $p = F_z / A$ weight: $F_z = m \cdot g$ mass: $m = \rho \cdot V$

pressure: $p = F_z / A =$

Osmotic pressure



$$\Pi = \rho \cdot g \cdot h$$

At very low concentration:

$\rho = \rho_{solution} \approx \rho_{solvent}$

so in approximation:

$$\Pi = \rho_{solvent} \cdot g \cdot h$$

and earlier we saw: $\Pi = [B]RT = \frac{n_B}{V}RT$

Answers:

Question 1

a) $\Delta T = \left(\frac{RT_{trs}^{*2}}{\Delta_{trs}H}\right) x_B$ and x_B= (#mole Ag^+ + #mole Cl^-) / (#mole H_2O + #mole Ag^+ + #mole Cl^-)

= ($1.31 \cdot 10^{-5} \cdot 2$) / (997/18.015 + $1.31 \cdot 10^{-5} \cdot 2$) = $4.73 \cdot 10^{-7}$

ΔT = (8.3145 · 273.15^2/$6.008 \cdot 10^3$) · $4.73 \cdot 10^{-7}$ = **4.89×10^{-5} K**

the approximation: $x_B \approx$ (#mole Ag^+ + #mole Cl^-) / (#mole H_2O) also gives ΔT = 4.89×10^{-5} K as an answer

b) $x_B = \Delta T \left(\frac{\Delta_{trs}H}{RT_{trs}^{*2}}\right)$

= 1.00 · ($6.008 \cdot 10^3$/8.3145 · 273.15^2) = $9.68 \cdot 10^{-3}$

x_B = #mole DMSO/(#mole H_2O + #mole DMSO)

x_B = #mole DMSO /(997/18.015 + #mole DMSO) =>

x_B (55.34 + #mole DMSO) = #mole DMSO => #mole DMSO = 0.536/(1 - x_B)

#mole DMSO = 0.536/(1 - $9.68 \cdot 10^{-3}$) = 0.541

m_{DMSO} = 0.541·78.13 = **42.3 g**

the approximation: $x_B \approx$ #mole DMSO/ #mole H_2O gives m_{DMSO} = 41.9 g as an answer

Question 2

$\prod = \rho g h = [B]\mathrm{R}T$ approximation: $\rho_{\mathrm{solute}} \approx \rho_{\mathrm{water}}$

$[B] = \rho g h/(\mathrm{R}T) = 0.997 \cdot 10^3 \cdot 9.81 \cdot 5.746 \cdot 10^{-2} /(8.3145 \cdot 293) = 0.2307\ \mathrm{mol/m^3}$

$= 0.2307\ \mu\mathrm{mol/cm^3} = 0.2307 \cdot 10^{-6}\ \mathrm{mol/cm^3}$

$M_{\mathrm{enzyme}} = m_{\mathrm{enzyme}} / n_{\mathrm{enzyme}} = 3.221 \cdot 10^{-3}\ \mathrm{g} / 0.2307 \cdot 10^{-6}\ \mathrm{mol} =$ 13.96×10^3 g/mole

Question 3

$x_{\mathrm{B}} = \Delta T \left(\frac{\Delta_{trs} H}{\mathrm{R} T_{trs}^{*2}}\right) = 0.780 \cdot (18.80 \cdot 10^3 / 8.3145 \cdot 354^2) = 1.407 \cdot 10^{-2}$

x_{B} = #mole solute/(#mole solute + #mole solvent)

#mole solvent = $m_{\mathrm{solvent}} / M_{\mathrm{solvent}}$ = 250 / 128.18 = 1.95

x_{B} = #mole solute /(#mole solute + 1.95) =>

$1.407 \cdot 10^{-2}$(#mole solute + 1.95) = #mole solute =>

#mole solute = $0.02745/(1 - 1.407 \cdot 10^{-2})$ = 0.02784

$M_{\mathrm{solute}} = m_{\mathrm{solute}} / n_{\mathrm{solute}}$ = 5.00 / 0.02784 = 179.6 = 180 g/mole

the approximation: $x_{\mathrm{B}} \approx$ #mole solute/#mole solvent gives M_{solute} = 182 g/mole as an answer